In conclusion, it was found in this experimental study of the density distribution in the hypersonic viscous layer that the scaling parameter λ_f correlates the shock strength data remarkably well from the kinetic into the strong interaction region, and that the numerical results based on the Rubin-Lin theory agree well with the experimental data for $X \geqq 15\lambda_f$.

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Effect of Axisymmetric Imperfections on the Vibrations of Cylindrical Shells under Axial Compression

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Nomenclature

C = coefficient in Eq. (5)
c = $[3(1-\nu^2)]^{1/2}$
E = Young's modulus
F = stress function
f = frequency
h = shell thickness
l = shell length
m = mass per unit area
N_x = axial membrane stress resultant
N_y = circumferential membrane stress resultant
N_{xy} = shear membrane stress resultant
n = circumferential wave number
p = axial wave number
R = radius of middle surface of perfect cylindrical shell
T = $(4\rho^4+1)/(4\rho^4+1-4\lambda\rho^2)$
t = time
W = radial displacement (positive outward)
W_o = axisymmetric initial imperfection
w = vibration mode (or buckling mode when frequency is zero)
x = axial coordinate
y = circumferential coordinate
μ = parameter which determines amplitude of imperfection
ν = Poisson's ratio
λ = load parameter, $[3(1-\nu^2)]^{1/2}\sigma R/Eh$
σ = axial compressive stress
ρ^2 = $\{1/[3(1-\nu^2)]^{1/2}\}(h/R)p^2$
ϕ = stress function of additional stress due to vibration (or buckling when frequency is zero)
ω = circular frequency = $2\pi f$
Ω = frequency parameter = $(2\bar{R}^2m\omega^2/Eh)^{1/2}$
Ω_{min} = lowest frequency parameter for different values of τ at certain load
$(\ )_{,d}$ = differentiation with respect to d
$(\dot{\ })$ = differentiation with respect to time
Δ = Laplacian
τ^2 = $\{1/[3(1-\nu^2)]^{1/2}\}(h/R)n^2$

1. Introduction

SINCE the pioneering work of Koiter in 1945,[1] many studies have emphasized the important role that geometrical imperfections play in the buckling of shells. In particular, cylindrical shells under axial compression have been investigated extensively and the initial geometrical imperfections have been identified there as cause of the large discrepancies between the experimental buckling loads and the theoretical predictions. The present state-of-the-art is that, in most cases, low experimental buckling loads can qualitatively be attributed to initial geometrical imperfections. However, for better quantitative assessments, the nature of the initial imperfections has to be precisely known. From an engineering point of view, it is hence very important to know the precise imperfections of a certain shell or, preferably, the influence of these imperfections without actually buckling the shell, in order to predict the buckling load more accurately. The direct approach of measurement of the deviation of the shape of the structure from its ideal shape has been used in some studies.[2-4] This method is, however, complicated and requires sophisticated equipment and relatively long measurement times. In the present paper, an indirect approach is employed. It is shown that geometrical imperfections of the kind which affect buckling have also a large influence on the vibrations of these shells, even at zero load. This phenomenon facilitates measurement of the actual imperfections by measurement of the deviations in the frequencies of the imperfect shell from those of the corresponding perfect one.

2. Theory

The theoretical development is based on an analysis derived by Koiter.[5] As a matter of fact, the present study essentially only adds the inertia terms to Koiter's formulation. A detailed derivation of the equations is given in Ref. 11, here only the salient features are outlined.

As in Koiter's paper, only axisymmetric imperfections are considered. The equations of the nonlinear theory of shallow shells are employed, and they become in the present case

$$N_x = F_{,yy}, \quad N_y = F_{,xx}, \quad N_{xy} = -F_{,xy} \tag{1}$$

$$(1/Eh)\Delta\Delta F - (1/R)W_{,xx} + W_{o,xx}W_{,yy} + W_{,xx}W_{,yy} - (W_{,xy})^2 = 0 \tag{2}$$

$$[Eh^3/12(1-\nu^2)]\Delta\Delta W + (1/R)F_{,xx} - W_{o,xx}F_{,yy} - W_{,xx}F_{,yy} - W_{,yy}F_{,xx} + 2W_{,xy}F_{,xy} + m\ddot{w} = 0 \tag{3}$$

These equations appear in Koiter's paper[5] without the inertia term of Eq. (3) and are attributed there to Vol'mir.[6] Similar equations were used earlier[7] in the analysis of the nonlinear vibrations of cylindrical shells without consideration of imperfections. Only radial inertia is included here, whereas inplane inertia is neglected. However it was already shown earlier,[8] that for isotropic cylindrical shells consideration of radial inertia only, yields very good results if compared with more exact analysis.

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Following Koiter the basic axisymmetric case is first solved. Since this state is static the results are the same, as in Ref. 5. A simple form of axisymmetric initial imperfection is assumed

$$W_o(x) = -\mu h \cos(2px/R) \tag{4}$$

Then small asymmetric vibrations are superposed on the basic axisymmetric state. These small superposed vibrations are considered to be infinitesimal and hence the equations may be linearized with respect to them, as in Ref. 5. Only the first approximation of the assumed asymmetric mode of vibration is considered here. Hence

$$w_{(x,y,t)} = Ch \cos(px/R)\cos(ny/R)\, e^{i\omega t} \tag{5}$$

Some restrictions on boundary conditions and length are implied by this solution. These are discussed in Refs. 5 and 11, and are shown to be negligible in practice.

Then the two linearized equations obtained from Eqs. (2) and (3) are solved. The first is solved directly, and the second approximately, by the Galerkin method. One then obtains an expression for the frequencies of different modes of vibration of simply supported cylindrical shells as a function of the axial load and the magnitude of the axisymmetric imperfection. This expression is

$$2\Omega^2 = (\rho^2+\tau^2)^2 + 4\rho^4(\rho^2+\tau^2)^{-2} - 4\rho^2\lambda - 2c\mu\tau^2[T-1+ 8T\rho^4(\rho^2+\tau^2)^{-2}] + 16(c\mu)^2T^2\rho^4\tau^4[(\rho^2+\tau^2)^{-2} + (9\rho^2+\tau^2)^{-2}] \tag{6}$$

One of the definitions of the buckling load is the load at which the structure exhibits for the first time a zero natural frequency. The mode of this vibration with zero frequency is also the buckling mode. By this criterion, Eq. (6) with $\Omega = 0$ yields the buckling load. Indeed, for vanishing Ω, one obtains

$$(\rho^2+\tau^2)^2 + 4\rho^4(\rho^2+\tau^2)^{-2} - 4\lambda\rho^2 - 2c\mu\tau^2[T-1+ 8T\rho^4(\rho^2+\tau^2)^{-2}] + 16(c\mu)^2T^2\rho^4\tau^4[(\rho^2+\tau^2)^{-2} + (9\rho^2+\tau^2)^{-2}] = 0 \tag{7}$$

which is identical to the corresponding Eq. (5.2) of Ref. 5.

3. Results and Discussion

The influence of the imperfections on the frequency will now be studied. Assuming a certain mode of vibration, one may calculate the frequency of this mode and determine how it is affected by the imperfection. Following Koiter, the mode $\rho^2 = \tau^2 = \frac{1}{2}$, which is the buckling mode of the perfect cylinder, is chosen. For ease of correlation with Koiter's results, the curves are computed for $\nu = 0.272$, yielding, as in Ref. 5, $c = \frac{5}{3}$. In Fig. 1, one observes that for small μ, the frequency decreases with load till buckling. For larger amplitudes of imperfections μ, however, the frequency decreases at first with load for small loads but then, at higher loads, the phenomenon is reversed and the frequency rises with load without buckling. These results agree with those of Koiter,[5] (see Fig. 3 of Ref. 11), and were attributed by him to the stabilizing effect of the different geometry of the imperfect shell.

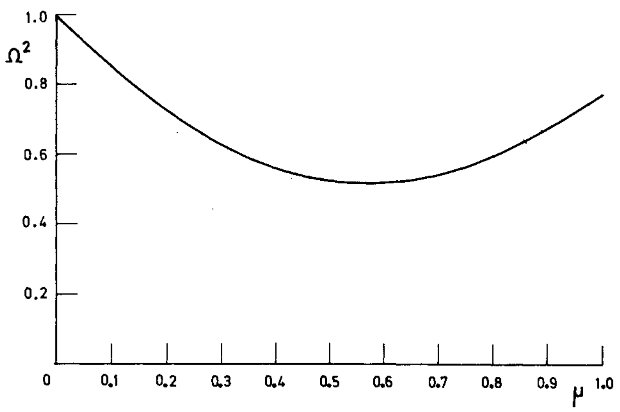

Fig. 2 Frequency parameter Ω^2 vs imperfection amplitude at zero load ($\rho^2 = \tau^2 = \frac{1}{2}$).

Now detection and evaluation of the initial imperfections at zero load is of utmost practical significance. In Fig. 2, the change in the frequency vs imperfection amplitude is plotted for the mode $\rho^2 = \tau^2 = \frac{1}{2}$ at zero load [represented also by Eq. (25) of Ref. 11]. It is immediately seen that the frequency decreases with increase in imperfection amplitude μ down to a minimum of 0.519 at $\mu = 0.577$, then Ω^2 increases again. This bears much similarity to the behavior of the special theory buckling load, for the same mode, given in Fig. 1 of Ref. 5. There, too, at $\mu = 0.579$, the curve reverses its direction.

Up to this point in the discussion only the mode $\rho^2 = \tau^2 = \frac{1}{2}$ was considered. The behavior of other circumferential modes will now be studied at zero load, but with the axial mode remaining at $\rho^2 = \frac{1}{2}$. Figure 3 shows the frequency parameter vs the circumferential mode shape τ^2 at zero load for different amplitudes of the imperfection μ. One observes that with increasing imperfection amplitude μ, the minimum frequency becomes smaller, and it happens in a mode with a smaller number of circumferential waves. In Fig. 4, this minimum frequency parameter $\Omega_{\min}{}^2$ is plotted as a function of μ and the corresponding values of τ^2 are indicated on the curve. [This curve is plotted with an approximate form of the frequency equation, Eq. (27) of Ref. 11, but it is shown there that the error due to this approximation is small.]

4. Conclusions

The study indicates that initial imperfections, which influence the buckling of cylindrical shells considerably, also have a large effect on the vibrations of these shells, even at zero axial load. In the present investigation, only the first approximation of an axisymmetric initial imperfection was considered. The analysis should be extended to higher approximations of axisymmetric

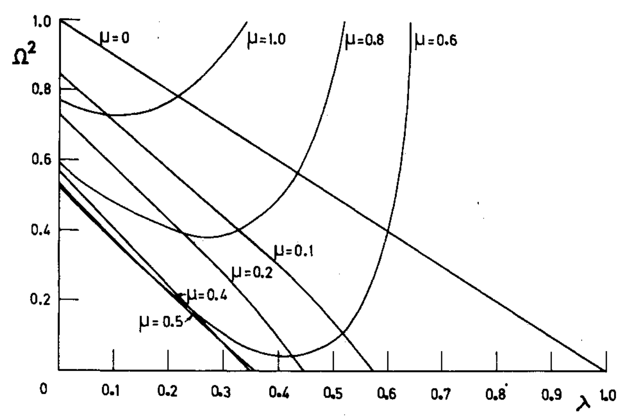

Fig. 1 Changes of frequency parameter Ω^2 vs load parameter λ for different values of imperfection amplitude μ. (Mode shape is $\rho^2 = \tau^2 = \frac{1}{2}$.)

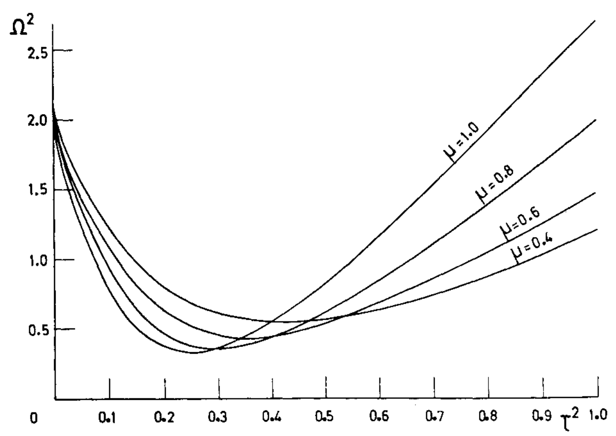

Fig. 3 Frequency parameter Ω^2 vs circumferential mode shape τ^2 at zero load for different amplitudes of imperfections ($\rho^2 = \frac{1}{2}$).

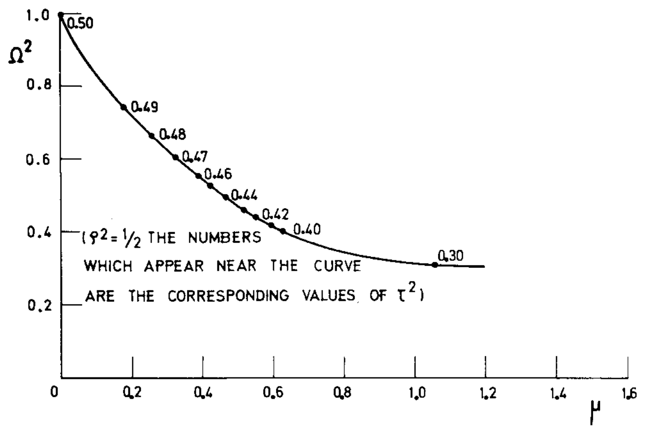

Fig. 4 Minimum frequency parameter vs amplitude of imperfections at zero load.

imperfections, and an even more important extension would be the study of the effect of the more practical asymmetric initial imperfections. Another useful direction of study would be the inclusion of other kinds of loading, such as hydrostatic pressure or torsion.

The authors believe that the problem of interaction between vibration and initial imperfections may also be approached in another manner. Such an energetic approach will essentially be an extension of the general theory of elastic stability developed by Koiter.[1] Koiter's general theory has been applied to nonlinear vibrations of beams and plates,[12] but without consideration of imperfections.

Of major practical importance is the feasibility—indicated in the present study—of obtaining an assessment of the initial imperfections in the shell by simply vibrating it and measuring the natural frequencies at zero or low loads. From previous experiments,[9,10] it is known that the vibration modes which can be detected accurately and conveniently are those connected with one or, at most, two axial half waves. In such cases, the boundary conditions have great influence on the vibrations and must be taken into account.

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Unsteadiness Measurements in the Flow Through an Oblique Shock Wave

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Nomenclature

$\tilde{e}$ = hot-wire rms fluctuation voltage
Δe_m = hot-wire voltage sensitivity to mass flow
Δe_{T_o} = hot-wire voltage sensitivity to total temperature
M = flow mach number
r = ratio of hot-wire sensitivities $\Delta e_m/\Delta e_{T_o}$
$R_{\theta u}$ = correlation coefficient between entropy and velocity fluctuations
$R_{\theta\sigma}$ = correlation coefficient between entropy and sound-pressure fluctuations
$R_{u\sigma}$ = correlation coefficient between vorticity and sound-pressure fluctuations
T_o = flow total temperature
$\tilde{u}$ = normalized rms longitudinal velocity fluctuation corresponding to the vorticity mode
v_* = virtual total temperature fluctuation $\tilde{e}/\Delta e_{T_o}$
α = Mach number function $\{1+[(\gamma-1)/2]M^2\}^{-1}$
β = Mach number function $\alpha(\gamma-1)M^2$
γ = ratio of the specific heats
$\tilde{\sigma}$ = normalized rms mass flow fluctuation corresponding to the sound-wave mode
$\tilde{\theta}$ = normalized rms total temperature fluctuation corresponding to the entropy mode

A RECENT work typified discussions on the interrelation between flow unsteadiness and aerodynamic measurements.[1] It concludes that a measure of the unsteadiness may be as relevant to some types of model tests as the Mach and Reynolds numbers, examples of which are boundary-layer transition experiments, buffet and flutter tests. Questions therefore arise concerning the measurement of the quantities which describe an unsteady compressible flow (vorticity, entropy, and sound-pressure fluctuations) and how to draw meaningful comparisons between data gathered by different types of sensors in different wind tunnels. A comparison of a limited number of data on pressure fluctuations in supersonic wind tunnels resolved by pressure transducers and hot-wire anemometers illustrated the "scatter" that can be expected.[2] Thus, are pressure transducer measurements (in which the transducer mounting body must shield the pressure sensitive element from part of the radiated pressure field) directly comparable with hot-wire resolved values (for which the wire response is wavelength and wave-front direction selective)? Furthermore, for a transducer mounted on a body which generates a finite strength leading edge wave, are the local fluctuations to which the transducer responds identical to the freestream fluctuations or are the latter modified on traversing

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the wave? An analysis of the flowfield produced by the oblique impingement of sound waves on a plane normal shock wave predicted vorticity as one product of the interaction[3]; thus one mode of disturbance may in theory give rise to another. Hot-wire experiments in the local hypersonic flow over a slender cone ($M = 17.5$, shock wave pressure ratio = 2.7) showed very little change in the percentage rms mass flow and total temperature fluctuations from the freestream to the shock layer, the predominant disturbance in both regions being a radiated pressure fluctuation.[4] This Note presents contrary data for the hypersonic flow over an inclined flat plate, hot-wire mode diagrams clearly showing that the incident freestream pressure fluctuations result in a mixed-mode disturbance field downstream of the leading edge shock wave.

Tests were conducted in the freejet flow of a gun tunnel at a freestream Mach number of 7.13 and a total pressure of 4.21×10^6 N/m^2. A constant temperature hot-wire anemometer was used to measure the freestream disturbances in the nozzle outlet plane, and in the two-dimensional flow behind the leading edge shock wave on an inclined flat plate set at incidences of 1°, 5°, and 14°. Local aerodynamic conditions were calculated from a knowledge of the model and flowfield geometry and the measured surface static pressure. Figure 1 is a shadowgraph of the flow (model leading edge located in the nozzle exit plane off the plate) in which the hot-wire is seen positioned midway between the surface and the leading edge wave with the model boundary layer adjacent to the wire of laminar form. Thus the sound-pressure field from the boundary layer was negligible and the disturbances to which the wire responded were those convected and radiated from upstream regions.

The freestream mode diagrams (virtual total temperature fluctuation v_* vs the ratio of wire sensitivities r) and the resulting percentage rms mass flow, total temperature, and pressure fluctuations are given in Fig. 2. Because of a falling stagnation, temperature gun tunnel flows are inherently nonstationary; therefore timewise data are presented, the first (50 msec) timing being dictated by the flow starting transients which must be allowed to subside and the necessary averaging time for the fluctuation signal. According to the Kovasznay analysis of hot-wire response in compressible flows[5] as extended by Laufer,[6] the linear mode diagrams show the wire responding to mass flow and total temperature fluctuations brought about by a train of "moving Mach waves" originating from turbulent eddies within the nozzle wall boundary layer. Therefore the freestream disturbance in the gun tunnel is a radiated pressure fluctuation as indeed is most frequently encountered in more conventional supersonic and hypersonic wind tunnels. The mean rms static pressure fluctuation is calculated as 2.7%.

The hot-wire mode diagrams for the flow downstream of the leading edge shock wave on the inclined flat plate are given in Fig. 3, in which it is seen that as the model incidence is increased (increasing shock strength) the mode diagrams assume a progressively more negative slope. The significance of this fact may be interpreted in the following manner. In Fig. 4 are illustrated the individual mode diagrams for the three independent

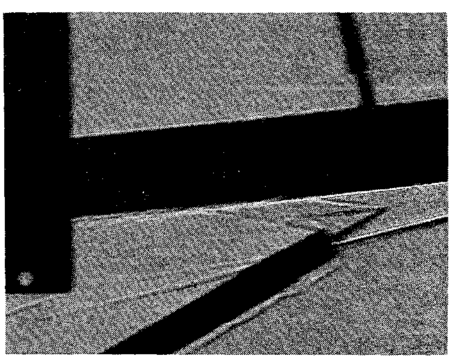

Fig. 1 Shadowgraph of the flow over the inclined flat plate.

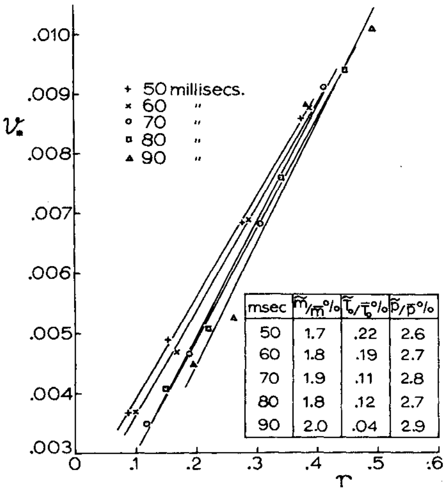

Fig. 2 Freestream hot-wire mode diagrams.

fluctuations, that is, vorticity, entropy, and sound-pressure. The mode diagram for all three fluctuations occurring together is given by the equation[5]:

$$v_*^2 = [(\alpha+r)/\alpha]^2\tilde{\theta}^2 + (\beta-r)^2\tilde{u}^2 + r^2\tilde{\sigma}^2 + 2[(\alpha+r)/\alpha](\beta-r)\tilde{\theta}\tilde{u}R_{\theta u} - 2[(\alpha+r)/\alpha]r\tilde{\theta}\tilde{\sigma}R_{\theta\sigma} - 2(\beta-r)r\tilde{u}\tilde{\sigma}R_{u\sigma} \quad (1)$$

If the three fluctuations are uncorrelated ($R_{\theta u} = R_{\theta\sigma} = R_{u\sigma} = 0$) the individual values of v_* add in the square, and it is clear from Fig. 4 that the general mode diagram for uncorrelated fluctuations can assume zero or negative slope in the range $r \leq \beta$ only when vorticity is present in the flow. On differentiating Eq. (1) further, mode diagrams having zero or negative slope are seen to be feasible, a general condition being that at least one of the correlation coefficients be nonzero with their necessary signs depending upon the values of the other parameters. Nonzero correlation coefficients clearly imply a fluctuation field containing at least two modes of disturbance. Referring now to the mode diagrams of Fig. 3 in which all curves display $dv_*/dr \leq 0$, it follows from the previous discussion that the flow downstream of the leading edge shock wave must contain a superposition of disturbances which may be either uncorrelated or correlated to

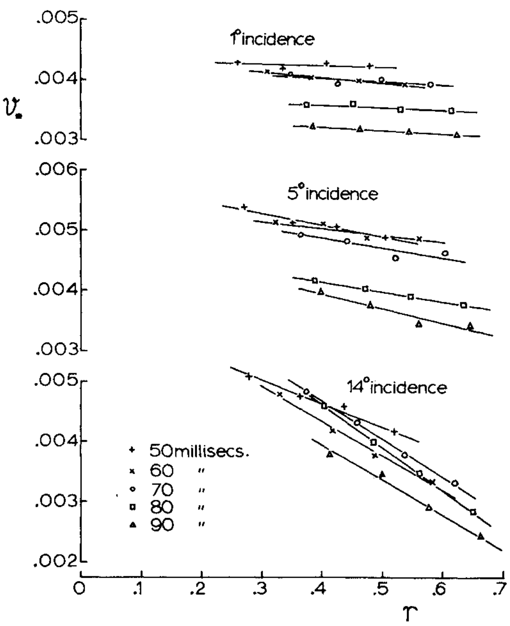

Fig. 3 Hot-wire mode diagrams for the flow over the inclined flat plate.

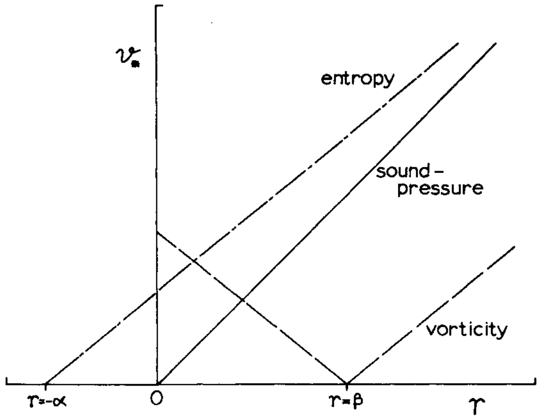

Fig. 4 Hot-wire mode diagrams for the three basic fluctuations.

varying degrees. Thus by means of hot-wire mode diagrams of the freestream and local model flows, it has been shown that in the gun tunnel a freestream sound-pressure fluctuation is "converted" into a multimode disturbance field on traversing an oblique shock wave.

This finding is in one sense consistent with a physical aspect of the current flow. The linearized theory of Kovasznay shows that the disturbance modes are noninteracting in a small perturbation field; that is, in the freestream and local model flows one mode may not give rise to another although they may coexist having derived from common or separate sources. Depending on its strength, within the plane of the shock wave the perturbations in the fluid properties may be large, which thereby allows for the possibility of the incident sound waves producing vorticity or entropy fluctuations at that location. Thus the shock wave becomes the "source" of the mixed disturbance field as detected by the hot-wire. Here mechanisms are not specified, although simple models may readily be postulated, in particular that for an incident pressure fluctuation giving rise to vorticity downstream of the shock wave.

High supersonic Mach number flows containing a superposition of significant disturbances including vorticity are difficult to analyze by means of hot-wire mode diagrams. Referring to Fig. 4, as $M \to \infty$, $\beta \to 2$, but even for modest hypersonic Mach numbers, say Mach 5, $\beta = 1.67$. Now the wire sensitivity ratio r is virtually synonymous with the wire overheating ratio, and for currently available high temperature wire alloys and for hypersonic flows with moderate stagnation temperatures ($T_o \sim 600°$K, for example) $r_{max} \sim 0.8$. Thus only a restricted range of r may be available over which must be determined the precise form of the mode diagram; moreover it is a curve which may have a minima at a value of r greater than the maximum achievable in practice. For higher temperature flows this restriction is even more severe. Gun tunnel flows, while being small perturbations, are inherently less steady than those generated in conventional wind tunnels, and scatter in the hot-wire data results. Therefore in the current tests no attempt was made to obtain sufficient data for the numerical resolution of the multimode flow over the inclined flat plate. Nevertheless, the least squares curves through four data points presented in Fig. 3 are of a form which clearly supports the main argument of this Note.

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Unsteady Flow Downstream of an Airfoil Oscillating in a Supersonic Stream

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Introduction

EXISTING closed form solutions for linearized, two-dimensional, supersonic flows past oscillating airfoils neglect the influence of the unsteady wake on the flowfield.[1] In such cases, it has still been possible to obtain the relevant information desired from solutions (i.e., airfoil pressure distributions and aerodynamic forces and moments) since the wakes only affect the flowfield downstream of the airfoils. Recently, the problem of the supersonic flow with subsonic axial velocity component past an oscillating cascade of airfoils has received serious attention.[2–5] For this flow geometry, the unsteady wakes influence the flowfield adjacent to blade surfaces and, hence, the wakes must be included in the analysis. This factor has posed a formidable obstacle in attempts to derive an exact solution and to date only approximate results have been obtained. With the purpose of providing further insight into this and other unsteady supersonic flow problems in which unsteady wakes must be considered, the flow past an oscillating airfoil (Fig. 1) has been re-examined. The solution in the region bounded by the airfoil leading and trailing edge Mach waves has been obtained previously by several investigators.[1,6] The solution for the unsteady flowfield downstream of the trailing edge Mach waves is described in this Note.

Analysis

All parameters discussed below are dimensionless. Lengths have been scaled with respect to blade chord, time with respect to blade chord divided by the freestream speed, and pressure with respect to the freestream density multiplied by one-half of the square of the freestream speed. The airfoil is assumed to be a flat plate performing rapid harmonic motions of small amplitude generally normal to the stream direction. These motions occur at a prescribed frequency ω. The unsteady wake is a thin vortex sheet which emanates from the trailing edge of the airfoil and extends infinitely far downstream. The small amplitude assumption permits the equations governing the unsteady flowfield to be linearized and leads to the additional simplification that boundary conditions, which apply on the airfoil and wake surfaces, can be satisfied on the mean position of these surfaces.

A modified velocity potential, $\psi(x, y)$, determined by the relation

$$\psi(x, y) = \phi(x, y, t) \exp\left[i(kMx - \omega t)\right] \tag{1}$$

is introduced to simplify the equations governing the unsteady flow. Here ϕ is the velocity potential as defined in the usual manner, $k = \omega M \mu^{-2}$, and M is the freestream Mach number. The modified potential must satisfy the differential equation[7]

$$\psi_{yy} - \mu^2 \psi_{xx} - \mu^2 k^2 \psi = 0 \tag{2}$$

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